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# Shapes of Cosmic Strings and Baryon Number Violation

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## Abstract

The relation between shapes of cosmic string and baryon number violation is investigated. If there exist fermionic zero modes on the string, using bosonization technique, it is possible to obtain the effective action which describes the fermion coupled to the arbitrarily shaped cosmic string. The relation between baryon number and the sum of the writhing number and linking number of the cosmic strings is rederived. Baryons are created on the strings as the shapes of the cosmic strings change. Furthermore we discuss implications of this baryon number violating process to baryogenesis.

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# 1 Introduction

Toward a better understanding of the physics on the string-like objects, it is important to search for typical phenomena of their structures of extent. It is especially interesting to investigate the entanglement of the string-like objects, which is inherent in the string-like objects in four dimensional space-time. Recently the present author and Yahikozawa attempted to find such phenomena for the Nielsen-Olesen vortices [1]. It was found that the expectation value of  $\int dx^4 F_{\mu\nu} \tilde{F}_{\mu\nu}$  under the background of the Nielsen-Olesen vortices becomes the difference between the sum of the linking number and the writhing number of them at the initial time and the one at the final time. This means that if chiral fermions are coupled to the vortices, fermion number is violated as vortices change their shapes, because of the anomaly. In this letter we include fermion in the system explicitly and closely examine this mechanism of fermion number violation.

Another purpose of this letter is related to baryogenesis. Baryogenesis has become a topic of much recent activity. Not to contradict with inflation scenarios, it is preferred that a baryon asymmetry is produced at lower temperature. In many GUT theories, the reheating temperature after inflation must be lower than GUT scale [2]-[5]. This infers that baryogenesis at GUT scale [6]-[10] is incompatible with inflation. To consider alternative scenarios of baryogenesis, we must introduce a baryon number violating process at lower than GUT scale without contradiction to proton decay experiments. In most alternative baryogenesis scenarios, the EW sphaleron [11] was used [12, 13, 14]. Our mechanism of fermion number violation is another way to introduce a baryon number violating process without contradiction to proton decay experiments. We also investigate implications of our fermion number violating process to baryogenesis.

There are related papers to this work. The relation between the linking number of the strings and baryon number was discussed in [15, 16, 17]. However, they neglected the contribution from the writhing number of the strings. Inclusion of fermion into the system was attempted in [17], but an only restricted configuration of the strings was considered.

In the following, we will construct the effective action of the fermion coupled to *arbitrarily shaped* strings and show explicitly that baryons and leptons are created on the strings as they change their shapes. Furthermore we discuss its implications to baryogenesis. Units in which  $k_B = \hbar = c = 1$  will be used and  $m_{\text{Pl}}$  is the Planck mass.

## 2 The construction of effective action of fermion coupled to arbitrarily shaped strings

The model which we will consider is

$$\begin{aligned}
\mathcal{L} &= \mathcal{L}_{gauge-Higgs} + \mathcal{L}_{fermion}, \\
\mathcal{L}_{gauge-Higgs} &= -\frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} + D_\mu\varphi^*D^\mu\varphi - \lambda(|\varphi|^2 - v^2)^2, \\
\mathcal{L}_{fermion} &= \bar{U}_L i \not{D} U_L + \bar{U}_R i \not{D} U_R - y_q(\bar{U}_R\varphi U_L + \text{h.c.}) \\
&\quad + \bar{E}_L i \not{D} E_L + \bar{E}_R i \not{D} E_R - y_l(\bar{E}_R\varphi^* E_L + \text{h.c.}),
\end{aligned} \tag{1}$$

where  $Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$  and  $D_\mu\varphi = (\partial_\mu - iA_\mu)\varphi$ . The  $U(1)$  charges are assigned as follows.

matter	$U_R$	$U_L$	$E_R$	$E_L$	$\varphi$
charge	$r + 1/2$	$r - 1/2$	$\hat{r} - 1/2$	$\hat{r} + 1/2$	$+1$

Table I.  $U(1)$  charge assignment

The anomaly cancellation condition requires that  $r$  and  $\hat{r}$  must satisfy the relation  $r^2 = \hat{r}^2$ . Because of the anomaly, the ‘‘baryon’’ number current  $j_B^\mu \equiv \bar{U}\gamma^\mu U$  and the ‘‘lepton’’ number current  $j_L^\mu \equiv \bar{E}\gamma^\mu E$  do not conserve and satisfy the following relations.

$$\begin{aligned}
\partial_\mu j_B^\mu &= -\frac{r}{8\pi^2} \tilde{Z}^{\mu\nu} Z_{\mu\nu}, \\
\partial_\mu j_L^\mu &= \frac{\hat{r}}{8\pi^2} \tilde{Z}^{\mu\nu} Z_{\mu\nu},
\end{aligned} \tag{2}$$

where  $\tilde{Z}^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma} Z_{\rho\sigma}/2$  and  $\epsilon^{0123} = -\epsilon_{0123} = 1$ . From these equation, we obtain

$$\begin{aligned}
Q_B &= -\frac{r}{8\pi^2} \epsilon_{ijk} \int d^3x Z_i Z_{jk}, \\
Q_L &= \frac{\hat{r}}{8\pi^2} \epsilon_{ijk} \int d^3x Z_i Z_{jk}.
\end{aligned} \tag{3}$$

When the  $U(1)$  symmetry is spontaneously broken, cosmic strings form. Under the background of the strings, the flux is concentrated on the strings and this can be expressed as

$$Z_{\mu\nu}(x) = -\frac{1}{4} \epsilon_{\mu\nu\rho\sigma} J^{\rho\sigma}(x), \tag{4}$$

where

$$J^{\mu\nu}(x) = 4\pi \sum_I \int d^2\tau \frac{\partial X_I^{[\mu}}{\partial \tau^0} \frac{\partial X_I^{\nu]}}{\partial \tau^1} \delta^4(x - X_I(\tau)). \tag{5}$$

Here  $X_I^\mu$  denotes the position of the I-th cosmic string and  $\tau$  is the coordinate parameterizing the world sheet swept by the cosmic string. Solving this equation under the gauge conditions  $\tau^0 = X_I^0$  and  $\partial_i Z_i = 0$ ,  $Z_\mu$  become

$$\begin{aligned} Z_0(\mathbf{x}, t) &= -\frac{1}{2}\epsilon_{ijk} \sum_I \oint d\tau^1 \frac{\partial X_I^i}{\partial \tau^1} \frac{\partial X_I^j}{\partial t} \partial_k \left( \frac{1}{|\mathbf{x} - \mathbf{X}_I|} \right), \\ Z_i(\mathbf{x}, t) &= -\frac{1}{2}\epsilon_{ijk} \sum_I \oint d\tau^1 \frac{\partial X_I^j}{\partial \tau^1} \partial_k \left( \frac{1}{|\mathbf{x} - \mathbf{X}_I|} \right). \end{aligned} \quad (6)$$

Using this and (2), we obtain

$$\begin{aligned} Q_B &= -r \left( \sum_I W_r(\mathbf{X}_I) + \sum_{I \neq J} L_k(\mathbf{X}_I, \mathbf{X}_J) \right), \\ Q_L &= \hat{r} \left( \sum_I W_r(\mathbf{X}_I) + \sum_{I \neq J} L_k(\mathbf{X}_I, \mathbf{X}_J) \right). \end{aligned} \quad (7)$$

The definition of  $W_r$  and  $L_k$  are

$$\begin{aligned} W_r(\mathbf{X}) &= \frac{1}{4\pi} \oint d\tau_1 \oint d\tau'_1 \epsilon_{ijk} \frac{\partial X^i}{\partial \tau_1} \frac{\partial X^j}{\partial \tau'_1} \frac{(X(\tau) - X(\tau'))^k}{|\mathbf{X}(\tau) - \mathbf{X}(\tau')|^3}, \\ L_k(\mathbf{X}, \mathbf{Y}) &= \frac{1}{4\pi} \oint d\tau_1 \oint d\tau'_1 \epsilon_{ijk} \frac{\partial X^i}{\partial \tau_1} \frac{\partial Y^j}{\partial \tau'_1} \frac{(X(\tau) - Y(\tau'))^k}{|\mathbf{X}(\tau) - \mathbf{Y}(\tau')|^3}. \end{aligned} \quad (8)$$

$L_k$  is the Gauss linking number and it takes an integer.  $W_k$  is called the writhing number and in general not takes integer [18]. This is a geometrical quantity and measures the twist of the cosmic string. Unless the cosmic string intersects with itself, the value of the writhing number changes continuously as the shapes of the string change. When the string intersects with itself, the value of the writhing number changes by 2. An explicit example of the writhing number was given in [19].

From (7), it is found that the “baryon” number and the “lepton” number change according to the shapes of cosmic strings. To examine this more closely, we will analysis the fermionic part of the system.

On the cosmic string, the vacuum expectation value of the Higgs field  $\varphi$  becomes zero. Since the fermions get masses by the Yukawa coupling, massless fermionic modes exist on the string. In the following, we will derive the effective action of these massless modes and show that these massless modes are produced on the strings as “baryon” or “lepton”.

First we will derive the effective action of the massless modes on the z-directed cosmic string. The field configuration of  $\varphi$  and  $Z_\mu$  are given as the Nielsen-Olesen solution  $\varphi^{\text{vortex}}$  and

$Z_\mu^{\text{vortex}}$  [20]. On this background, it is known that there exists a solution of the Dirac equation which is independent on  $z$  and  $t$  and normalized in the  $x$ - $y$  plane [21, 22]. We denote this as  $(U_L^\dot{\alpha}, U_{R\alpha}, E_L^\dot{\alpha}, E_{R\alpha}) = (\beta_L^\dot{\alpha}(x, y), \beta_{R\alpha}(x, y), \hat{\beta}_L^\dot{\alpha}(x, y), \hat{\beta}_{R\alpha}(x, y))$ .  $\beta$  and  $\hat{\beta}$  satisfy  $\sigma_3\beta_R = \beta_R$ ,  $\sigma_3\beta_L = -\beta_L$ ,  $\sigma_3\hat{\beta}_R = -\hat{\beta}_R$  and  $\sigma_3\hat{\beta}_L = \hat{\beta}_L$ . We take the normalization as  $\int dx dy |\beta|^2 = 1/2$ . Using this solution we can express the fermion constrained on the string as

$$\begin{cases} U_L^\dot{\alpha}(x, y, z, t) = q(z, t)\beta_L^\dot{\alpha}(x, y) \\ U_{R\alpha}(x, y, z, t) = q(z, t)\beta_{R\alpha}(x, y) \end{cases} \quad (9)$$

$$\begin{cases} E_L^\dot{\alpha}(x, y, z, t) = l(z, t)\hat{\beta}_L^\dot{\alpha}(x, y) \\ E_{R\alpha}(x, y, z, t) = l(z, t)\hat{\beta}_{R\alpha}(x, y) \end{cases} \quad (10)$$

Substituting these configuration for  $\mathcal{L}_{\text{fermion}}$  and integrating over  $x$  and  $y$ , we obtain

$$L_{\text{eff}} = iq^*(\partial_0 - irz_0)q + iq^*(\partial_3 - irz_3)q + il^*(\partial_0 - i\hat{r}z_0)l - il^*(\partial_3 - i\hat{r}z_3)l, \quad (11)$$

where we define  $L_{\text{eff}}$  as  $\int dx dy \mathcal{L}_{\text{fermion}}$ . The gauge fields  $z_a$  are  $\mu = 0, 3$  components of the  $Z_\mu^{\text{vortex}}$ . For the  $z$ -directed string they can be zero, but for arbitrarily shaped strings they are not zero.

If we use  $q_r \equiv (q, 0)^T$  and  $l_l \equiv (0, l)^T$ , this effective lagrangian can be rewritten as the following more familiar form

$$L_{\text{eff}} = i\bar{q}_r\gamma^a(\partial_a - irz_a)q_r + i\bar{l}_l\gamma^a(\partial_a - i\hat{r}z_a)l_l, \quad (12)$$

where  $a = 0, 3$  and the 2-dimensional gamma matrix is defined as  $\gamma^0 = \sigma^1$  and  $\gamma^3 = -i\sigma^2$ .

The ‘‘baryon’’ number current and the ‘‘lepton’’ number current on the string can be obtained in the similar manner. These are defined as  $J_B^a = \int dx dy j_B^{\mu=a}$  and  $J_L^a = \int dx dy j_L^{\mu=a}$  and become

$$\begin{aligned} J_B^a &= \bar{q}_r\gamma^a q_r, \\ J_L^a &= \bar{l}_l\gamma^a l_l. \end{aligned} \quad (13)$$

To discuss the effective action on the arbitrarily shaped string, bosonization technique is useful, since in the bosonized fields the anomaly can be treated classically [23, 24]. Bosonization rules are given by  $i\bar{q}\gamma^a\partial_a q \rightarrow \frac{1}{2}\partial_a\phi_q\partial^a\phi_q$ ,  $\bar{q}\gamma^a q \rightarrow \frac{1}{\sqrt{\pi}}\epsilon^{ab}\partial_b\phi_q$  [25, 26]. Using the relation  $\bar{q}\gamma^a\gamma_5 q = -\epsilon^{ab}\bar{q}\gamma_b q$ , we obtain

$$\begin{aligned} L_{\text{eff}} &= \frac{1}{2}\partial_a\phi_q\partial^a\phi_q + \frac{r}{2\sqrt{\pi}}(\epsilon^{ab} - \eta^{ab})z_a\partial_b\phi_q + \frac{r^2}{8\pi}z_az^a \\ &+ \frac{1}{2}\partial_a\phi_l\partial^a\phi_l + \frac{\hat{r}}{2\sqrt{\pi}}(\epsilon^{ab} + \eta^{ab})z_a\partial_b\phi_l + \frac{\hat{r}^2}{8\pi}z_az^a, \end{aligned} \quad (14)$$

and

$$\begin{aligned} J_B^a &= \frac{1}{2\sqrt{\pi}}(\epsilon^{ab} - \eta^{ab})\partial_b\phi_q - \frac{r}{4\pi}(\epsilon^{ab} - \eta^{ab})z_b, \\ J_L^a &= \frac{1}{2\sqrt{\pi}}(\epsilon^{ab} + \eta^{ab})\partial_b\phi_l + \frac{\hat{r}}{4\pi}(\epsilon^{ab} + \eta^{ab})z_b. \end{aligned} \quad (15)$$

Here  $\phi_q$  and  $\phi_l$  are bosonized fields of  $q$  and  $l$  respectively. The terms which do not depend on  $\phi_q$  or  $\phi_l$  are determined by gauge invariance. For the bosonization fields, gauge transformation can be defined as

$$\begin{aligned} z_a &\rightarrow z_a + \partial_a\Lambda, \\ \phi_q &\rightarrow \phi_q + \frac{r\Lambda}{2\sqrt{\pi}}, \\ \phi_l &\rightarrow \phi_l - \frac{\hat{r}\Lambda}{2\sqrt{\pi}}. \end{aligned} \quad (16)$$

This gauge transformation is expected from the operator relation of  $q$  and  $\phi_q$ , and only when the anomaly cancellation condition is satisfied the bosonized action becomes gauge invariant.

Now we will consider the effective action of the massless modes on an arbitrarily shaped string  $X^\mu(\tau)$ . This action is obtained from the above action by the replacement  $\eta^{ab} \rightarrow \sqrt{g}g^{ab}$  and  $(z, t) \rightarrow (\tau^0, \tau^1)$ , where  $g_{ab} = \partial_a X^\mu \partial_b X_\mu$  is the induced metric on the string and  $g = \det g_{ab}$ . The effective action is

$$\begin{aligned} L_{eff} &= \frac{1}{2}\sqrt{-g}g^{ab}\partial_a\phi_q\partial_b\phi_q + \frac{r}{2\sqrt{\pi}}(\epsilon^{ab} - \sqrt{-g}g^{ab})z_a\partial_b\phi_q + \frac{r^2}{8\pi}\sqrt{-g}g^{ab}z_az_b \\ &+ \frac{1}{2}\sqrt{-g}g^{ab}\partial_a\phi_l\partial_b\phi_l + \frac{\hat{r}}{2\sqrt{\pi}}(\epsilon^{ab} + \sqrt{-g}g^{ab})z_a\partial_b\phi_q + \frac{\hat{r}^2}{8\pi}\sqrt{-g}g^{ab}z_az_b \end{aligned} \quad (17)$$

and the “baryon” number current and the “lepton” number current on the arbitrarily shaped string are

$$\begin{aligned} \sqrt{-g}J_B^a &= \frac{1}{2\sqrt{\pi}}(\epsilon^{ab} - \sqrt{-g}g^{ab})\partial_b\phi_q - \frac{r}{4\pi}(\epsilon^{ab} - \sqrt{-g}g^{ab})z_b, \\ \sqrt{-g}J_L^a &= \frac{1}{2\sqrt{\pi}}(\epsilon^{ab} + \sqrt{-g}g^{ab})\partial_b\phi_l + \frac{\hat{r}}{4\pi}(\epsilon^{ab} + \sqrt{-g}g^{ab})z_b. \end{aligned} \quad (18)$$

Considering the massless modes on the I-th cosmic string,  $z_a(\tau)$  are given by  $z_a(\tau) = (\partial X_I^\mu / \partial \tau^a) Z_\mu^{\text{vortex}}(X(\tau))$ . When the radius of curvature of the string is much greater than the string thickness,  $Z_\mu^{\text{vortex}}$  are given as (6), therefore

$$\begin{aligned} z_0(\tau) &= -\frac{1}{2}\epsilon_{ijk} \sum_J \oint d\tau' \frac{\partial X_J^i}{\partial \tau'} \left( \frac{\partial X_I^j(\tau)}{\partial t} - \frac{\partial X_J^j(\tau')}{\partial t} \right) \frac{(X_I(\tau) - X_J(\tau'))^k}{|\mathbf{X}_I(\tau) - \mathbf{X}_J(\tau')|^3}, \\ z_1(\tau) &= \frac{1}{2}\epsilon_{ijk} \sum_J \oint d\tau' \frac{\partial X_I^i}{\partial \tau} \frac{\partial X_J^j}{\partial \tau'} \frac{(X_I(\tau) - X_J(\tau'))^k}{|\mathbf{X}_I(\tau) - \mathbf{X}_J(\tau')|^3}. \end{aligned} \quad (19)$$

Note that  $z_a(\tau)$  are not singular at  $\tau = \tau'$ . As far as the string does not intersect,  $z_a(\tau)$  are well-defined functions. This non-singularity of  $z_a$  is not depend on the gauge condition we take. Even if we use  $\partial_\mu Z^\mu = 0$  gauge, they are not singular. This enables us to tell what occurs on the string definitely and makes the effective action (17) useful.

Using the equation of motion, we obtain

$$\begin{aligned}\partial_a \sqrt{-g} J_B^a &= -\frac{r}{4\pi} \epsilon^{ab} z_{ab}, \\ \partial_a \sqrt{-g} J_L^a &= \frac{\hat{r}}{4\pi} \epsilon^{ab} z_{ab}.\end{aligned}\tag{20}$$

From (19), we find that the “baryon” number on the string  $Q_B^{(2)} \equiv \oint d\tau^1 \sqrt{-g} J_B^a$  and the “lepton” number on the string  $Q_L^{(2)} \equiv \oint d\tau^1 \sqrt{-g} J_L^a$  satisfy

$$\begin{aligned}\Delta Q_B^{(2)} &= -r \Delta \left( W_r(\mathbf{X}_I) + \sum_J L_k(\mathbf{X}_I, \mathbf{X}_J) \right), \\ \Delta Q_L^{(2)} &= \hat{r} \Delta \left( W_r(\mathbf{X}_I) + \sum_J L_k(\mathbf{X}_I, \mathbf{X}_J) \right).\end{aligned}\tag{21}$$

These relations are consistent with (7) and show that “baryons” and “leptons” are produced on the strings according to the changes of their shapes.

Although we have treated the only toy model in this section, the conclusions do not change for more complicated models. If all the fermions get masses by Yukawa coupling, the effective action of the massless modes can be constructed in the same manner and it is found that baryons and leptons are produced on the string as the cosmic strings change their shapes. Indeed in the cases of the electroweak string [28, 29] and cosmic strings in supersymmetric model with an extra  $U(1)$  [31], we have constructed the effective actions of the string-fermion system except the neutrino part [27]. If massless fermions exist, the above analysis do not apply in general, however, since (7) is irrelevant to the fermion mass, we believe that the conclusions do not change in this case.

### 3 Implications to baryogenesis

In the previous section, we have shown that as cosmic strings change their shapes, baryons are generated on them. In the following we will discuss baryogenesis due to this baryon number violation mechanism.

We will consider cosmic strings on which baryons are generated by our mechanism of baryon number violation. For example, the electroweak string is this type of the cosmic strings. Another example is cosmic strings in a supersymmetric model with an extra  $U(1)$ .

As first pointed out by Sakharov, three conditions need to be satisfied in order to generate a net baryon number [32]. First, baryon number violating process must exist. Second, these process must violate C and CP. And third, they must occur out of thermal equilibrium. C and CP are violated in usual particle physics models and once cosmic strings form, they become rapidly out of equilibrium. Therefore we assume that second and third conditions are satisfied. In the following we will concentrate on the first condition.

It is expected that our mechanism of baryon number violation is the most efficient at string formation. If cosmic strings form by the Kibble mechanism [33], the correlation length of the strings  $\xi$  at formation is given by  $\xi(\eta) \sim (\lambda\eta)^{-1}$ , where  $\eta$  is the scale of the symmetry breaking by which cosmic string forms and  $\lambda$  is a Higgs self coupling constant or a gauge coupling constant. Since the strings form by randomly assigning values of the phase of the Higgs field, it is expected that the string twists once per  $(\lambda\eta)^{-3}$  volumes. Therefore the number of baryons or anti-baryons created on the strings is about  $(\lambda\eta)^3$  particles per unit volume. This is comparable to the entropy of the massless particles at  $T = \eta$ .

Although there exists a baryon number violating process, it is not obvious to generate a net baryon number because the baryon number is violated due to the anomaly. In this case we find that a linear combination of the baryon number and the Chern-Simons form preserves. This gives a restriction to generate a net baryon number. For example, in our toy model,

$$Q_B + \frac{r}{8\pi^2} \epsilon_{ijk} \int dx^3 Z_i Z_{jk} \quad (22)$$

preserves. This tells us that if there is no baryon before cosmic strings form, no net baryon remains after cosmic strings vanish. This is because in the toy model cosmic strings form by spontaneous breaking of  $U(1)$  gauge symmetry. Since  $U(1)$  gauge symmetry has only one vacuum, no net baryon number is produced by the anomaly after cosmic strings vanish.

One possibility to obtain a net baryon number is to combine with an extra baryon number violating process. A candidate of the extra baryon number violating process is the sphaleron (or instanton) transition. An explicit baryon number violating process in GUT is another candidate, but if we consider baryogenesis scenarios after inflation, the sphaleron transition is more attractive since it does not contradict with experiments of the proton decay.

We assume that the extra baryon number violating process exists and the rate of the process is high at string formation. We further assume that after  $T = \eta'$ , this process becomes



suppressed because of a phase transition. Until  $T = \eta'$ , the baryon number produced by the strings is erased by the extra baryon number violating process. Thus  $Q_B$  in (22) is zero at  $T = \eta'$ . After  $T = \eta'$ , the linear combination of the baryon number and the Chern-Simon form (22) preserves, because the extra baryon number violating process becomes irrelevant. Therefore we find that a net baryon number remains after the strings vanish and this is given by the sum of the writhing number and the linking number of the strings at  $T = \eta'$ .

To estimate the sum of the writhing number at  $T = \eta'$ , considerations about the evolution of the string is needed. After formation, strings experience a significant damping force from the high plasma background density. A heuristic argument [33] show that in the friction dominated epoch the correlation length  $\xi$  change as

$$|\xi(T) - \xi(\eta)| \sim \left( \frac{m_{\text{Pl}}(\eta^2 - T^2)}{T^5} \right)^{1/2}. \quad (23)$$

This infers that the structures of the string smaller than  $(m_{\text{Pl}}/\eta^3)^{1/2}$  are straightened when  $T$  becomes slightly lower than  $\eta$ . This result is confirmed by solving the equation of motion. We take the metric of the Friedman universe as  $ds^2 = a^2(\tau)(d\tau^2 - d\mathbf{x}^2)$ , where  $\tau$  is conformal time  $d\tau = dt/a(t)$ . In the gauge conditions  $\tau_0 = \tau$  and  $\dot{\mathbf{x}} \cdot \mathbf{x}' = 0$ , the string evolves as

$$\ddot{\mathbf{x}} - \epsilon^{-1} \left( \frac{\mathbf{x}'}{\epsilon} \right)' + \left( 2\frac{\dot{a}}{a} + \frac{\beta T^3}{\mu} a \right) (1 - \dot{\mathbf{x}}^2) \dot{\mathbf{x}} = 0, \quad (24)$$

where  $\epsilon$  is given by

$$\epsilon = \left( \frac{\mathbf{x}'^2}{1 - \dot{\mathbf{x}}^2} \right)^{1/2} \quad (25)$$

and  $\mu \equiv \eta^2$  is the string tension and  $\beta$  is numerical constant of order unity [34]. Here dots and primes denote derivatives with respect  $\tau_0$  and  $\tau_1$ . Let us examine small perturbations on the static straight string. If we represent  $\mathbf{x}$  as  $\mathbf{x} = \mathbf{e}\tau_1 + \delta\mathbf{x}(\tau_0)e^{ik\tau_1}$ , where  $\mathbf{e}$  is an unit vector, and neglect the Hubble damping term, the equation of motion becomes

$$\delta\ddot{\mathbf{x}} + \frac{\beta T^3}{\mu} a \delta\dot{\mathbf{x}} + k^2 \delta\mathbf{x} = 0. \quad (26)$$

In the radiation era,  $a$  and  $T$  are given as  $a \sim m_{\text{Pl}}\tau_0$  and  $T \sim 2/\tau_0$ . In order to examine this equation at  $\tau_0 \sim \eta^{-1}$ , it is sufficient to solve the equation where the coefficient of  $\delta\dot{\mathbf{x}}$  is replaced with a constant  $\beta m_{\text{Pl}}$ . The solution can be easily obtained and we find that if  $k/a$  is larger than  $(\eta^3/m_{\text{Pl}})^{1/2}$ , the amplitude of  $\delta\mathbf{x}$  is damped within  $\Delta\tau_0 \sim 1/\eta$ . This means that structures of the strings smaller than  $(m_{\text{Pl}}/\eta^3)^{1/2}$  are straightened within  $\Delta T \sim \eta'$ . This result agrees with the heuristic argument<sup>1</sup>.

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<sup>1</sup> Same conclusion was obtained in [35, 36].

We have found that a net baryon obtained is determined by the sum of writhing number and the linking number of the strings at  $T = \eta'$ . The sum of the writhing number per unit volume is expected to be  $\xi(\eta')^{-3}$ . If CP violation bias parameter is denoted as  $\varepsilon$ , we obtain a net baryon number per unit volume as  $\varepsilon \xi(\eta')^{-3}$ . Since the entropy of massless particles is given by  $s \sim T^3$ , the baryon to entropy ration will be

$$\frac{n_B}{s} \sim \varepsilon \left( \frac{\eta'^3}{m_{\text{Pl}}(\eta^2 - \eta'^2)} \right)^{3/2}. \quad (27)$$

This gives a severe constraint to work the scenario. Unless  $\eta$  is close to  $m_{\text{Pl}}$  or  $\varepsilon$  is very large, we find that  $\eta'$  must almost coincide with  $\eta$  to obtain a baryon asymmetry of the magnitude required to explain the present baryon to entropy ratio. If we consider cosmic string forming at  $O(\text{TeV})$  and the EW sphaleron as the extra baryon number violating process, it is impossible to obtain the baryon asymmetry.

There is another possibility to obtain a net baryon number. Let us consider the string-forming phase transition  $G \rightarrow H$ . If  $\pi_1(G/H)$  is non-abelian, a topological argument tells that the strings corresponding to non-commuting elements of  $\pi_1$  cannot pass through each other [37, 38]. Therefore the linking number of the cosmic strings cannot almost change and a net baryon number remains. Vachaspati and Field estimated numerically that the linking number of the cosmic strings per unit volume is  $10^{-4}(\lambda\eta)^{-3}$  [16]. This gives the baryon to entropy ratio as  $10^{-4}\varepsilon\lambda^{-3}$ .

We comment the evolution of this type of strings. It was pointed out that this type of cosmic strings form a tangled network and if they form by the Kibble mechanism, they dominate the universe very soon after formation [39]. This is an obstacle to the baryogenesis scenario. However, the conclusion changes if we take into account intercommuting of the string at crossing itself. Since an element of  $\pi_1$  commutes with itself, the topological argument gives no constraint when the string crosses itself. Then this intercommuting is possible. This enables not to form a tangled string network, since by intercommuting, tangled long strings become loosely linked long strings with many linked tiny strings. Therefore too early string-domination in universe does not occur.

## 4 Conclusions

In this letter we have presented the effective action of fermion coupled to arbitrarily shaped strings. As was suggested in [1], the fermion are created as the strings change their shapes.

Furthermore we have discussed its implication to baryogenesis. We have considered two possibilities to obtain a net baryon number. The first is to combine with an extra baryon number violating process. Considered carefully the evolution of cosmic strings in the friction dominated epoch, it has been shown that string-forming scale must be very close to the scale where the extra baryon number violating process begins to suppress. This is because the structures of the strings smaller than  $(m_{\text{Pl}}/\eta^3)^{1/2}$  are straighten within  $\Delta T \sim \eta$ . This constraint is not inherent in our baryon number violating process. Defect-mediated electroweak baryogenesis [30, 31] also suffers this constraint <sup>2</sup>. As a second possibility, we have considered the cosmic string with non-abelian  $\pi_1(G/H)$ . It is possible to produce a baryon asymmetry without too early string-domination in universe.

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<sup>2</sup>Eq.(3.10) in [30] is not correct and their conclusion must be changed.

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